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Non-local percolation and partial ordering in the dilute Baxter–Wu model

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Abstract. The zero-temperature limit of the dilute Baxter-Wu model is studied in the context of percolation theory. The percolation model describing the propagation of order is identified and shown to possess a non-local mechanism for the propagation of that order. A Monte Carlo study of this model is performed using the non-local cluster identification algorithm developed by Fried and Schick. We also show that the mechanism responsible for non-local percolation allows for the possibility of the propagation of partial order. At intermediate impurity concentration a partially ordered phase is found and its structure is discussed.

1. Introduction

The present work is devoted to a study of the quench-diluted Baxter-Wu model at zero temperature. The Baxter-Wu model is well known for its exact solution found in the absence of dilution by Baxter and Wu [1]. Subsequently Novotny and Landau [2] performed a finite-temperature Monte Carlo study of the quench-diluted model and verified the validity of the Harris criterion [3]. (Since the exact solution yields $\alpha = \frac{2}{3} > 0$, the Harris criterion implies that the disorder is relevant—the result found by Novotny and Landau.) In the present context we investigate the model's zero-temperature structure by employing the well known relationship between the zero-temperature structure of quench-diluted lattice spin models (i.e. a magnetic model with quenched non-magnetic impurities) and percolation models [4]. For this relationship the concentration of magnetic spins is interpreted as the concentration of occupied sites in the associated percolation model and is denoted by the site occupation probability $0 \leqslant p \leqslant 1$. The percolation clusters are defined as those clusters of spins which, by the constraint of energy minimization at zero temperature, must support an ordering in one of the degenerate ground states of the pure model (see figure 1).

Our principal result is to identify the appropriate percolation model characterizing the structure of the dilute Baxter-Wu model at zero temperature and to show that the resulting model exhibits a novel non-local mechanism for the propagation order. Recently it has been shown that the dilute triangular antiferromagnetic $q = 3$ Potts (DTA3P) model [5] and the rigidity percolation model [6] also share this key feature of non-locality.

An example of a percolation model exhibiting only local properties is the geometric percolation model [7] which is related to the zero-temperature dilute Ising ferromagnet [4]. For this model the fundamental constraint for the propagation of order is that

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two occupied lattice sites α and β are constrained to the same percolation cluster if these sites are connected by a pairwise bond $b_{\alpha,\beta}$. (The allowed bonds follow from the particular form chosen for the Ising Hamiltonian.) In magnetic language, this constraint means that when the state of the spin (here $S_\alpha = +1$ or -1) at site α is specified, the state at site β is uniquely determined and the ordered state (in this cas e ferromagnetic) is propagated between the two sites. This mechanism of a pairwise constraint is local in that the complete complement of information necessary for the propagation of order from site α to site β is localized on site α. Therefore the geometric percolation model is characterized as a local percolation model.

In contrast, a percolation model can exhibit non-local properties when the fundamental constraint employed in the propagation of the ordered state is of insufficient strength to guarantee that a complete ordering is propagated, though in general it can guarantee that at least a partial ordering propagates. An implication of this partial propagation of order is that though complete order can still be propagated, the constraints responsible for this propagation are now found to be delocalized over arbitrarily large regions of the lattice. This delocalization of the constraints necessary for the propagation of order characterizes the zero-temperature dilute Baxter–Wu model and related percolation models as non-local percolation models.

A dramatic consequence of this non-local percolation mechanism is that the propagation of order between two distinct regions of the lattice may be mediated by regions that do not completely share the propagated order [5, 8]. In section 4 the consequences of this emergent property are explored for the Baxter–Wu model and at intermediate impurity concentrations a zero-temperature phase is found which lacks long-range order, yet exhibits a partial ordering. This partially ordered phase is of the same general form as that exhibited by the DTA3P model [5] and its generalizations [9]. These models were introduced to help in characterizing the percolation properties of the solid $(ortho\text{-}H_2)_p(para\text{-}H_2)_{1-p}$ system [10] (the spherical *para*-H_2 molecules play the role of impurities) and may therefore be thought of as discrete state analogoues of this dilute quadrupolar system. It is interesting to note that for intermediate *para*-H_2 concentrations, Harris and Meyer [10] have observed the onset of large relaxation times at low temperature, though they argue that random-field effects imply that no finite-temperature phase transition to a quadrupolar glass phase is expected. It is also interesting to note that for the rigidity percolation model, Wang and Harris [11] have found what they term 'a splay-rigid phase', exhibiting vanishing bulk and shear moduli and a non-zero Frank elastic constant. This splay-rigid phase can be thought of as the analogue to our partially ordered phase.

The non-local cluster identification algorithm developed by Fried and Schick [8] is employed in Monte Carlo calculations [12] to study the quantitative properties of the Baxter–Wu percolation model. This algorithm was developed in response to the observation by Adler *et al* [5] that non-local percolation mechanisms violate the assumptions of the Hoshen–Kopelman (local) cluster identification algorithm [13]. A non-local algorithm has also been developed for the rigidity percolation model [6]. In this case, however, the connection rules underlying the cluster identification do not form a closed set and additional rules must be added at each level of iteration. In our algorithm, the hierarchy of connection rules truncates after two levels of iteration and so allows an exact determination of the non-local percolation clusters.

This paper is organized as follows: in section 2 we define the dilute Baxter–Wu model, in section 3 we expose the percolation mechanism responsible for the non-local propagation of order at zero temperature, discuss the associated cluster identification

algorithm and present the results of Monte Carlo calculations utilizing this algorithm. In section 4 we discuss the qualitative character of the partially ordered phase found at intermediate impurity concentrations.

2. The dilute Baxter-Wu model

The Hamiltonian for the dilute Baxter-Wu model [2] has the form

$$H = -J \sum_{\langle ijk \rangle} n_i n_j n_k S_i S_j S_k. \tag{1}$$

The sum is taken over all elementary triangles of the triangular lattice (i.e. all triangles pointing up and down composed of three mutual nearest neighbours). $J > 0$ is a three-spin coupling and the $S_i = \pm 1$ are Ising spins located at the vertices of the triangular lattice. There are no pairwise interactions between the Ising spins. The site occupation variables, n_i, have the value $n_i = 0$ if the site is vacant (i.e. occupied by a non-magnetic impurity) and $n_i = 1$ if it is occupied by an Ising spin. The sites of the triangular lattice are taken to be occupied at random with concentration $p = \langle n_i \rangle$. Since the site occupation for this model is random, we may write the statistical weight for a given realization $\{n_i\}$ of occupied sites as

$$W(\{n_i\}, p) = p^{n_s}(1-p)^{N-n_s} \tag{1a}$$

where N is the total number of sites on the lattice and $n_s = \Sigma_i n_i$ is the total number of occupied sites. We call this a random or uncorrelated percolation model, a characteristic it shares with such models as the geometric [7], bootstrap [14, 15] and directed [16, 17] percolation models. In contrast, when the statistical weight $W(\{n_i\}, p)$ for agiven realization $\{n_i\}$ is no longer simply a product of statistically independent site probabilities, then such a model is called a correlated percolation model [18]. Examples of this latter class of models are: the q-state Potts 'thermally' correlated percolation model [19] (the bond formulation of this model has been studied in detail by Hu [20]), the spin-glass model introduced by Adler *et al* [21] and the class of quasirandom models studied by Fried and Schick [9].

It is known from the solution of Baxter and Wu [1] that for $p = 1$ a phase transition occurs, at a temperature T_c, from a high-temperature disordered phase to a low-temperature phase which occupies one of the four ordered states shown in figure 1. One state corresponds to the ferromagnetic state (all $S_i = 1$) and the three remaining ordered states correspond to placing $S_i = 1$ on only one of the three sublattices labelled 1, 2, 3. To characterize the structure of the dilute model there are two specific questions to consider: the first concerns the location of the critical concentration p_c below which long-range order of the type exhibited by the pure model becomes impossible. Here one assumes that the transition temperature $T_c(p)$ decreases as p decreases from unity and for some critical concentration p_c, $T_c(p_c) = 0$. The second question concerns the location of the critical concentration p_c^e below which the presence of quenched dilution causes the system to fragment into a collection energetically decoupled of finite-size subsystems. When this decoupling condition is met the free energy for a specific (though average) configuration of dilution takes the form

$$F = \sum_i F_i. \tag{2}$$

The $\{F_i\}$ are the free-energy contributions from the statistically independent finite-size subsystems, which by virtue of their finite size are non-singular at all temperatures.

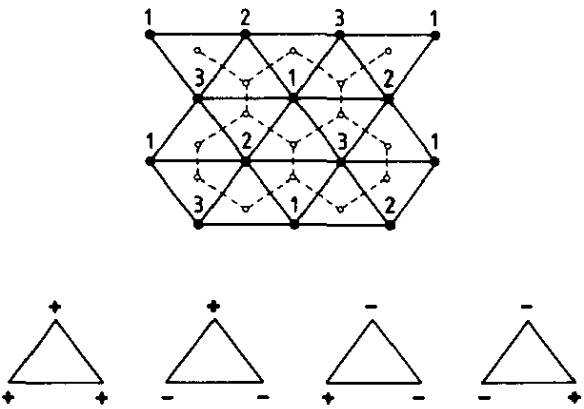

Figure 1. Here we show a portion of the triangular lattice with its dual, the honeycomb lattice, formed by connecting the centres of the triangles. Below are shown the four ordered states of the pure Baxter–Wu model. The values of the spin states shown at the vertices of the triangles represent the values occupying a given sublattice on the triangular lattice. These three sublattices are labelled 1, 2, 3 on the vertices of the triangular lattice.

This implies that F is non-singular and by implication, no phase transition can occur for $p < p_c^e$. The dilute nearest-neighbour ferromagnetic Ising model is a special case with $p_c = p_c^e$. For an answer to these questions it is sufficient to consider the zero-temperature limit because both p_c and p_c^e characterize 'configurational' properties which are not modified at finite temperature since the location of the impurities is quenched. We, therefore, turn to a discussion of the percolation models relevant to this zero-temperature structure.

3. A non-local percolation model

In this section we identify the percolation model which characterizes the propagation of the ordered state in the dilute Baxter–Wu model at zero temperature. As discussed in the previous section, this percolation model exhibits a percolation transition located at a critical concentration p_c. We also briefly describe the associated percolation cluster identification algorithm employed in the Monte Carlo study of this percolation model and present the results of this study.

To begin, note that any fully occupied triangle (pointing either up or down) of the triangular lattice must, since $J > 0$, lower its energy by occupying one of the four degenerate ground states depicted in figure 1. This implies that a fully occupied triangle is the smallest unit on the lattice capable of supporting a well defined ordering. Furthermore, as the three-spin interaction only acts on these triangles, the propagation of order will only involve the mutual constraints of energy minimization acting between these fully occupied triangles. For example, in figure 2(a), the triangle pointing up is fixed in one of the ordered states. This leaves a single spin of the triangle pointing down (the one enclosed by the square) undetermined. That this remaining occupied site must be in the state shown follows trivially from the constraint of energy minimization. A consequence of this result is that fully occupied triangles, which share an edge, must be contained in the same ordered state and so, by definition, the sites comprising these two triangles are contained in (or coalesce into) the same percolation cluster. For a given realization of occupied sites $\{n_i\}$, we can therefore identify those

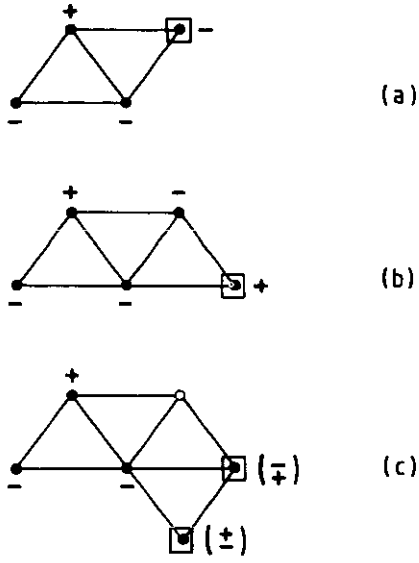

Figure 2. Here we show the three basic configurations of sites relevant to the non-local percolation model. Occupied sites are represented by full circles; vacant sites by open circles. (*a*) Two fully occupied triangles that share an edge. The left-hand triangle is placed in the state shown, the site enclosed by the square is constrained to the state shown as discussed in the text. This configuration constitutes the basic mechanism for the local rule for the propagation of order. (*b*) Three fully occupied triangles; the left-hand triangle is placed in the state shown and via the local rule order propagates to the site enclosed by the square. (*c*) A configuration of four triangles, with one site vacant, leaving two fully occupied triangles. That ordered is incompletely propagated is denoted by the multiple values that the two sites enclosed by squares may occupy.

percolation clusters $\{C_i\}$ which are formed when we employ only this local rule (i.e. shared edge between neighbouring triangles). In figure 2(*b*) we show, using this local rule, the ordered state propagating from the triangle on the left-hand side, to the central triangle (as in figure 2(*a*)), and on to the triangle located on the right-hand side.

If the local rule were the only means of propagating the ordered state then the set of percolation clusters $\{C_i\}$ would constitute an exact enumeration of the percolation clusters embodied by the realization $\{n_i\}$. To see that the local clusters $\{C_i\}$ do not represent the exact percolation clusters, consider the configuration shown in figure 2(*c*). For this configuration, since only two of the four triangles shown are fully occupied (the open circle represents a vacant site), only these can have spin states constrained by the three-spin interaction. With the left-hand triangle placed in the ordered state shown, only the single site shared by both the left- and right-hand occupied triangles is constrained to a well defined spin state; two of the sites (both of which are enclosed by squares) remain undetermined. Note, however, that due to the form of the three-spin interaction, the state of the right-hand triangle is not completely undetermined. The maximum fourfold degeneracy between the fixed state of the left-hand triangle and the allowable states of the right-hand triangle has reduced to a twofold degeneracy. Therefore, though complete order has not been propagated between the two triangles, a partial ordering has propagated. The possibility of partial ordering is the primary mechanism responsible for the non-local propagation of the ordered state.

We formulate the general constraint for the complete propagation of order between clusters as the *general 'connection' rule*; two clusters C_1 and C_2 coalesce into the same percolation cluster C_{12}, when they share sites on at least two of the three sublattices of the triangular lattice (see figure 1). In figure 2 we have shown the elementary examples illustrating this rule. For a more complex example consider the site configuration shown in figure 3(a). This configuration consists of two locally defined clusters; one to the left and one to the right of the site enclosed by the open square. As the two clusters share only one site, then with the left-hand cluster placed in the state shown, there remains a twofold degeneracy in the ordering of the right-hand cluster. In this case the two clusters are distinct. In figure 3(b), again there are two locally defined clusters which share two sites on distinct sublattices. It follows by application of the general rule that the ordered state is propagated between these clusters so they coalesce to form a single cluster. The important point to note is that the constraints necessary for the propagation of the order between these two clusters are not localized to any single triangle of occupied sites. These constraints have been delocalized over the two clusters and now involve essentially all of the sites that compose the two clusters. This condition is exemplified by the fact that the two sites enclosed by squares (i.e. those 'connections' which are primarily responsible for communicating the order) can occur at arbitrary separations from one another. The mechanism for the propagation of order is therefore non-local in character.

To appreciate the level of complexity that arises from these non-local constraints, assume that for a given realization $\{n_i\}$ we have identified the local clusters $\{C_i\}$. These clusters form the 'backbone' for the non-local percolation clusters $\{\tilde{C}_i\}$. In figure 3 we have shown an example where two clusters share a pair of sites. It is also possible to

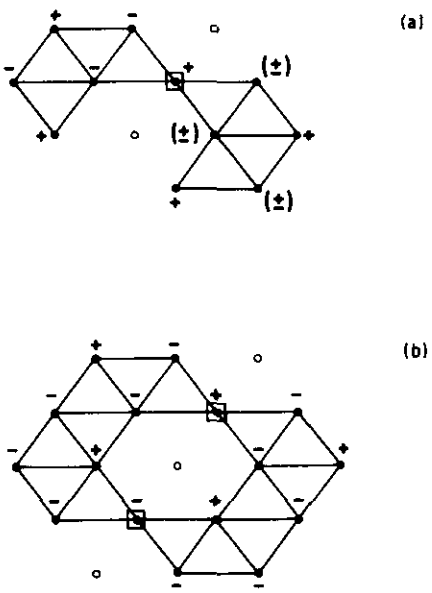

Figure 3. An example of the non-local propagation of order. (a) Here two locally defined clusters which share the site enclosed by the square. The left-hand cluster is placed in the state shown. That order does not propagate is denoted by multiple values, shown in parentheses, near the sites whose states are incompletely determined. (b) In this configuration, as the two clusters now share sites on two distinct sublattices, the conditions of the general connection rule are satisfied and complete order is propagated.

substitute an 'arbitrary region' capable of independently supporting an ordered state for either one or both of these sites. In figure 4 we illustrate an elementary (though typical) substitution configuration composed of three locally defined clusters C_{ab}, C_{ac} and C_{bc} labelled by the particular 'connection' sites they contain. In the absence of the connection point labelled c, the cluster C_{ab} could communicate only a partial ordering (see figure 2(c)) to the clusters C_{ac} and C_{bc}, since each share only a single site with C_{ab}. That the state of the cluster C_{ac} is in fact completely determined follows from the additional constraint imposed on this cluster by the site (labelled c) shared by clusters C_{ac} and C_{bc}. The shared site labelled b between clusters C_{ab} and C_{bc} constrains the sublattice of cluster C_{bc} on which site c is located. The partial ordering of C_{bc} is therefore sufficient to constrain one of the two undetermined sublattices of C_{ac}, and so, by the general rule, is completely determined. The important point here is that cluster C_{bc}, though essential for the propagation of order, remains only partially ordered relative to clusters C_{ab} and C_{ac}.

The idea which underlies the implementation of the non-local algorithm is to identify intermediary clusters such as C_{bc} and to replace the connection

$$C_{ab} \rightarrow C_{bc} \rightarrow C_{ac} \tag{3}$$

by a 'virtual connection'

$$C_{ab} \Rightarrow C_{ac}. \tag{4}$$

This procedure, therefore, reduces all sublattice constriants involving intermediary clusters to virtual constraints between pairs of clusters. In [8] a detailed discussion of the implementation of this algorithm is given. It is important to note here that these 'virtual' connections can represent intermediary configurations of arbitrary complexity and the resulting ordered and partially ordered structures will exhibit most unusual morphologies.

The non-local algorithm has been implemented on a computer and a Monte Carlo [12] study of the associated percolation model has been made. We have implemented the procedure using lattices with linear sizes $L = 12$, 24, 36, 48 and toroidal boundary

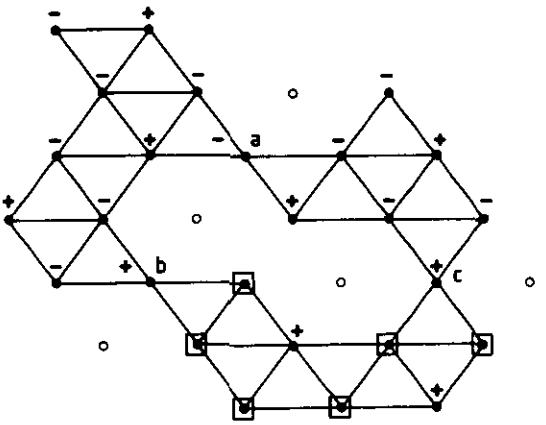

Figure 4. A site configuration is shown in which the order is propagated from the cluster C_{ab} to the cluster C_{ac}. The cluster C_{bc}, though essential for the propagation of this order, is itself not completely determined. (The subscripts on the cluster labels represent the sites contained in the specified cluster.)

conditions. For each distinct p-value, we have generated 1000 realizations $\{n_i\}$ for lattice sizes $L = 12$–36 and 100 realizations for $L = 48$. The renormalized site probability p' is defined by

$$p'(p, L) = \frac{N_p}{N_r}. \tag{5}$$

N_p is the number of realizations with at least one percolation cluster of linear size L and N_r is the total number of realizations generated. (Note that such a renormalization procedure corresponds to mapping the system of linear size L on to a single site.) The percolation order parameter $P(p, L)$ is defined by

$$P(p, L) = \frac{N_s(p, L)}{L^2}. \tag{6}$$

$N_s(p, L)$ is the average number of sites contained in the largest cluster on the lattice. Standard finite size scaling arguments [22] can be employed to extract values of the critical concentration and critical exponents characterizing the percolation transition. We find the specific values: $p_c = 0.755 \pm 0.01$, $\nu = 1.32 \pm 0.05$ and $\beta = 0.12 \pm 0.05$. These values for the critical exponents are numerically consistent with those of the geometric percolation model (with exact values [7]; $\nu = \frac{4}{3}$, $\beta = \frac{5}{36}$), a result that lends further support to recent arguments by Fried and Schick [9] concerning the critical properties of models of which this Baxter–Wu model is a special case.

In summary, we have defined the percolation model associated with the zero-temperature propagation of order in the dilute Baxter–Wu model. This percolation model has been shown to exhibit a non-local mechanism for the propagation of order. Monte Carlo calculations employing a non-local cluster identification algorithm have been performed and approximate values of the critical concentration p_c and critical exponents ν and β have been obtained. That non-local percolation structures allow the possibility of partial ordering has been noted. We now turn to a detailed consideration of the consequences of the propagation of this partial ordering.

4. A partially ordered phase

In the previous section we have evaluated the critical concentration p_c below which the system can no longer support long-range order of the kind found in the absence of dilution. In this section we focus on two topics: first, we show that there is an additional critical concentration $p_c^e < p_c$ characterizing the 'energetic fragmentation' of the system (as discussed in section 2) and we discuss the associated percolation model. Then we consider the structure of the zero-temperature phase that occupies the intermediate concentration range $p_c^e < p < p_c$, and show that it corresponds to a phase of partial order.

To begin, as shown in figure 1, the centres of the triangles on the triangular lattice form a honeycomb lattice. We define a prefacing transformation [9] from the triangular to the honeycomb lattice where a site of the honeycomb lattice is considered occupied when the three sites that enclose it are occupied. Since the three sites are occupied at random with probability p, the occupation probability on the honeycomb lattice is

$$p_\Delta = p^3. \tag{7}$$

The form of the three-spin interaction in equation (1) is such that two fully occupied triangles constrain one another energetically only if they share at least one site. This implies that energetic contact between fully occupied triangles extends to at most third neighbours with respect to the underlying honeycomb lattice (see figure 2(a)). Therefore under the prefacing transformation the 'energy-connectivity' percolation model appears to be equivalent to a first-, second- and third-neighbour geometric percolation model on a honecomb lattice. The critical concentration for this percolation model is known [4] to be $(p_\Delta)_c \approx 0.30$, and from equation (7) follows the prediction, $p_c^e \approx 0.67$. This calculation assumes, however, that under the prefacing transformation the site occupation on the honecomb lattice is random. However, this distribution is not random for a closely related model, as was pointed out by Fried and Schick [9]. The mechanism responsible for this non-random (or 'quasirandom') property is most easily seen in figure 2(b), where we note that the central triangle pointing down must be occupied when the two triangles pointing up are fully occupied. Therefore, under the prefacing transformation, two second-neighbour sites cannot both be occupied without their mutual nearest-neighbour site also being occupied. The analogous configuration with the central site vacant would be allowable, however, if the prefaced distribution were random. There are further constraints of this type that appear under the prefacing transformation, and when taken together, imply that the random distribution approximation places a lower bound to the critical concentration: $0.67 \leqslant p_c^e$. In their study of the dilute Baxter-Wu model, Novotny and Landau [2] (see also [23] for a similar calculation) implicitly account for these constraints by working directly on the triangular lattice where they found a value of $p_c^e = 0.71 \pm 0.01 < p_c$. This value for the energy-connectivity percolation threshold, as discussed in section 2, also constitutes a lower bound on p for the existence of phase transitions in the dilute Baxter-Wu model.

The critical concentration p_c^e not only signifies the onset of energy-connectivity percolation, but also represents a threshold for the percolation of partial order. This follows from the fact that the 'propagation' of partial order on the honeycomb lattice is mediated via occupied third neighbours (see figure 2(c)). This implies that for concentrations $p > p_c^e$, there is an extensive cluster of Ising spins whose allowable states, though correlated with one another, are not completely determined by one another (see figures 3 and 4). In contrast, for concentrations $p > p_c$ where the system exhibits long-range order, there is an extensive cluster of Ising spins constrained to occupy one of the four ordered states of the pure Baxter-Wu model. Therefore, the concentration range, $p_c^e < p < p_c$, denotes the boundaries of a phase exhibiting a partial ordering.

To explore the nature of this partially ordered phase, recall that in the previous section we identified the clusters $\{C_i\}$. These clusters were determined by application of the local constraint that propagates order only between fully occupied triangles which share an edge. The non-local clusters $\{\tilde{C}_i\}$ are defined by the application of the general rule to these local clusters. If we use the clusters $\{C_i\}$ to define a percolation model then we are led to consider a model previously studied in connection with the DTA3P models [5] and whose percolation threshold was found to be $p_c^l = 0.78 \pm 0.01$. (The superscript l denotes that we employ only the local clusters $\{C_i\}$.) It is at this threshold that there first occurs a cluster $C_p \in \{C_i\}$ which is extensive in size. We know that $p_c < p_c^l$, because the threshold p_c denotes the lowest concentration where an extensive cluster $\tilde{C}_i \in \{\tilde{C}_i\}$ first appears (the non-local constraints can only enhance the possibility of percolation beyond that exhibited by the purely local model). It is important to note that on the interval $p_c^e \leqslant p < p_c^l$, all of the clusters $\{C_i\}$ are of

finite size and so at $p = p_c$ no singular behaviour originates from those clusters. On the other hand, at this concentration, singular behaviour is exhibited by the clusters $\{\tilde{C}_i\}$ because this is the percolation threshold for the non-local model. On the interval $p_c^e \leq p < p_c$, we know that as long as we consider length scales $r < \xi$ (where $\xi \propto |p - p_c|^{-\nu}$ for $p \sim p_c$) the average structure of the model is indistinguishable from the large-scale structure in the range $p_c \leq p < p_c^l$. For $p \geq p_c^l$ this is no longer true because the long-range order is now supported by an extensive *locally* defined cluster, which, through it propagates the same ordered state, does so by a qualitatively different mechanism. The key point is that energy connectivity percolates for $p \geq p_c^e$ and the Baxter-Wu interaction enforces a partial ordering which introduces non-trivial correlations between the clusters $\{C_i\}$. This effect is clearly exhibited in the elementary example shown in figure 4. Therefore, throughout the interval $p_c^e \leq p < p_c$ on scales $r \sim \xi$, we see the system predominantly ordered in one of the four degenerate Baxter-Wu ordered states. In contrast, on scales $r > \xi$, we observe an amorphous or glassy structure.

In conclusion, we find that for concentrations $p < p_c$, the partially ordered phase exhibits a glassy structure. The mesoscopic morphology is characterized by domains of size $r \sim$ (few)ξ which are partially, though not completely, constrained by one another. For $p > p_c$, this glassy structure, though still present on the mesoscopic scale, is dominated by a single extensive percolating cluster supporting long-range order. Importantly, this percolating cluster is embedded within a background of partially ordered finite-sized clusters with whom it shares non-trivial correlations essential for the propagation of long-range order (e.g. figure 4). Recently, Adler *et al* [21] have introduced a class of competing interaction Ising models in which a zero-temperature partially ordered phase is realized upon the introduction of correlated dilution. The partially ordered phases of their models are, however, of the same general character as the one discussed here. It is also interesting to speculate that the partially ordered phase represents the zero-temperature limit of a glass phase reached via some finite-temperature glass transition (located at $T_g(p)$) from a high-temperature paramagnetic phase. The finite-temperature work of Landau and Novotny [2] did not consider concentrations small enough ($p \geq 0.778$) to study the question of a glassy phase directly. They did, however, find the interesting result: $T_c(p = 0.778)/T_c(p = 1) \approx 0.5$. This value seems quite large if one is to find $T_c(p_c) = 0$ even with account being made for finite size effects. Finite-temperature studies of a related model (the DTA3P model) are currently under way [24].

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